Rossby, Ertel, and potential vorticity

R. M. Samelson

College of Oceanic and Atmospheric Sciences

Oregon State University

104 Oc Adm Bldg

Corvallis, OR 97331-5503

USA

rsamelson@coas.oregonstate.edu

http://www.coas.oregonstate.edu/faculty/samelson.html

October 10, 2003

Abstract

It is frequently assumed that ROSSBY (1940) and ERTEL (1942a,b,c) derived their potential vorticity conservation laws independently. Two earlier publications (ROSSBY, 1936, 1938), a visit by Ertel to Rossby's department in 1937, and some other evidence make this traditional interpretation implausible.

1. Rossby

Carl-Gustav Rossby is most widely remembered for his many contributions to meteorology. His interests in what we would now call geophysical fluid dynamics, however, extended beyond the atmosphere to the ocean. In the course of analyzing a reduced-gravity shallow-water model of the Gulf Stream, ROSSBY (1936) derived the Lagrangian conservation law

$$f + \zeta = c\, D,$$

which holds exactly along fluid parcel trajectories in flow governed by the inviscid shallow water equations in a reference frame rotating at local angular velocity $f/2$ about the local vertical (where f may vary with position). Here ζ is the relative vorticity v_x - u_y (with u and v the x- and y-components of the horizontal velocity), D is the thickness of the (homogeneous) layer of fluid, and c is a constant that may depend upon the initial position of the fluid parcel. In this work, Rossby compared the conserved quantity represented by c to the Bernoulli function in steady flow, and showed how the local relative vorticity was related to the local thickness and the initial (resting) thickness by the conservation law.

ROSSBY (1938) generalized the potential vorticity conservation law to the case of a continuously stratified, hydrostatic fluid, again in the context of a theoretical study of ocean currents, by considering a stack of homogeneous fluid layers with infinitesimal thicknesses and density differences. This argument is repeated, in slightly different form, in ROSSBY (1940). The latter study of atmospheric flow patterns is frequently, and erroneously, cited as the original reference for Rossby's potential vorticity conservation law. The absence of references in ROSSBY (1940) to the earlier derivations by ROSSBY (1936, 1938) is one likely reason for this frequent error. ROSSBY (1940) does, however, mark the first appearance of the term "potential vorticity."

2. Ertel

By the early 1930's, Hans Ertel had begun to work on meteorology and vorticity dynamics. In 1933, for example, he published a new proof (ERTEL 1933) of the Bjerknes circulation theorem. In 1937, Ertel was invited to visit the Massachusetts Institute of Technology (MIT) and the Blue Hill Observatory for two months, to participate in an international isentropic analysis project (MAUERSBERGER 1971; PLATZMAN 1994; THORPE and VOLKERT 1997). The invitation evidently came from Rossby, who was chairman of the MIT Meteorological Division at that time. Jerome Namias, who also participated in the project, has written that Ertel "was a top-notch theoretician--and that is why Rossby invited him to MIT" (NAMIAS 1988). Despite the brevity of Ertel's stay at MIT, the preface of his well-received 1938 text (ERTEL 1938) lists the MIT Meteorological Division as his affiliation.

Five years later, in several concise and elegant contributions, Ertel derived a general set of potential vorticity theorems for ideal, non-hydrostatic, compressible fluids (ERTEL 1942a,b,c). These include, as a special case, an isentropic potential vorticity conservation law formulated in terms of potential temperature. Ertel's results are more general than Rossby's, and are stated and derived using vector calculus and a standard Eulerian vertical coordinate rather than the quasi-Lagrangian, isopycnal or isentropic layers and surfaces that Rossby used.

3. Discussion

In many contemporary articles and texts, it is suggested that Rossby and Ertel arrived at their potential vorticity conservation laws independently. The contact between Rossby and Ertel during Ertel's 1937 visit to MIT, and Rossby's 1936 and 1938 publications cited above, together make this traditional interpretation implausible. Another piece of circumstantial evidence, supporting the assumption that Ertel was by 1942 already aware of Rossby's work, is the reference to ROSSBY (1937), the companion paper to ROSSBY (1938), in ERTEL and JAW (1939).

It is worth noting here that Ertel's 1937 visit to MIT occurred between the publication dates of Rossby's original steady shallow-water potential vorticity (ROSSBY 1936) and his continuously stratified generalization (ROSSBY, 1938). Thus, it is possible that Ertel contributed directly to Rossby's generalization. However, since Rossby's generalization was based directly on a conceptual extension of the shallow-water layer model, while Ertel's approach and results are different and more general, the publication record does not provide any particular support for such an interpretation.

It is natural to ask why Ertel's 1942 articles do not cite Rossby's work, if Ertel had indeed been aware of it. This question cannot be decisively answered. It is possible that reference to prior work for the purposes of attribution was not of great concern to these authors; as noted above, Rossby had not himself bothered to cite ROSSBY (1936) or ROSSBY (1938) in the more well-known ROSSBY (1940). PLATZMAN (1994) and THORPE and VOLKERT (1997), respectively, point out the interesting facts that ERTEL (1942a,c) each contain an unusual, parenthetical reference to Rossby, in each case at the point where the isentropic potential vorticity is discussed. PLATZMAN (1994) suggested that the reference to Rossby in ERTEL (1942a) could be interpreted as an oblique acknowledgement of a familiarity with Rossby's potential vorticity. A literal translation of the relevant, final sentence of ERTEL (1942a), which exhibits an isentropic form of potential vorticity conservation very similar to Rossby's, is "The conservation law yields then...[eq. (12)], which relation [obtains meaning] for example in meteorology for the isentropic analysis (C. G. ROSSBY)." THORPE and VOLKERT (1997) give a translation of a similar sentence from ERTEL (1942c). In both of these sentences, the original German word-order places a verbal phrase (bracketed in the translation above) after the parentheses that end the translations, which has the effect of associating the reference to Rossby with the concept of isentropic analysis rather than with the conservation law. Nonetheless, Ertel may simply have felt that these parenthetical references were sufficient to indicate the connection between this special case of his results and the previous work in meteorology. PLATZMAN (1994) also noted that ERTEL (1942a,b,c) contain no references to articles published outside Germany, and that the political situation in 1942 may have made Ertel reluctant to include citations to work in languages other than German. In any case, it is hard to imagine that Ertel could have written these sentences without understanding the connection between his and Rossby's conservation principles.

There is no evidence that either Rossby or Ertel felt uncomfortable with the published record of their contributions to the development of the theory of potential vorticity conservation. The joint articles ERTEL and ROSSBY (1949a,b) might have offered convenient opportunities to amend the record, had they felt this necessary, but these cite only ERTEL (1942b,c). (Note here, however, that THORPE and VOLKERT (1997) speculate that ERTEL and ROSSBY (1949a,b) may have been written entirely by Ertel.) While the true history of their ideas remains uncertain, the circumstantial evidence is persuasive that Ertel was aware of Rossby's results on potential vorticity conservation well before 1942. Although Ertel's methods and approach were different than Rossby's, it is therefore difficult to escape the conclusion that the Ertel's thinking on this subject must have been influenced by Rossby's work. It seems equally clear that Rossby respected Ertel as a theoretician and regarded Ertel's results as substantial generalizations.

Acknowledgements This note owes a great debt to George W. Platzman, who first brought Ertel's 1937 visit to MIT to my attention. I am grateful to him for his comments and encouragement. I am grateful also to Alan Thorpe and Hans Volkert for their comments and for pointing out an omission in a previous version of this manuscript. Some of the present material previously appeared, in different form and without supporting citations, in SAMELSON (1998).

References

ERTEL, H. 1933: Ein neuer Beweis des hydrodynamischen Zirkulationstheorems. *Sitzungsberichte der Preussischen Akademie der Wissenschaften.* **XI**, 447-449.

ERTEL, H. 1938: *Methoden und Probleme der Dynamischen Meteorologie*. Ergebnisse der Mathematik und ihrer Grenzgebiete, 5. Band, Lieferung 3, Springer, Berlin, 153-278.

ERTEL, H. 1942a: Ein neuer hydrodynamischer Erhaltungssatz. *Die Naturwissenschaften*, **36**, 543-544.

ERTEL, H. 1942b: Über hydrodynamischer Wirbelsätze. *Physikalische Zeitschrift Leipzig*, **43**, 526-529.

ERTEL, H. 1942c: Ein neuer hydrodynamischer Wirbelsatz. *Meteorologische Zeitschrift*, **59**, 277-281.

ERTEL, H. and J.-J. JAW, 1939: Zur Integration der Rossbyschen Differentialgleichung der Antizyklogenesis. *Metorologische Zeitschrift*, **56**, 127-128.

ERTEL, H. and C.-G. ROSSBY, 1949a: Ein neuer Erhaltungs-Satz der Hyrodynamik. *Sitzungsberichte der Deutschen Akademie der Wissenschaften zu Berlin*, **I**, 3-11.

ERTEL, H. and C.-G. ROSSBY, 1949b: A new conservation-theorem of hydrodynamics. *Geofisica Pura e Applicata,* , **XIV**, 189-193.

MAUERSBERGER, P. 1971: Nachruf: Prof. Dr. Hans Ertel. *Zeitschrift für Meteorologie*, **22**, 314-328.

NAMIAS, J., 1988: Letter to G. Platzman, 29 July 1988, 2 pp.

PLATZMAN, G. W., 1994: Letter to R. Samelson, 1 September 1994, 3 pp.

ROSSBY, C.-G., 1936: Dynamics of steady ocean currents in the light of experimental fluid dynamics. *Papers in Physical Oceanography and Meteorology,* **5**, 1-43.

ROSSBY, C.-G., 1937: On the mutual adjustment of pressure and velocity distributions in certain simple current systems. *Journal of Marine Research,* **1**, 15-28.

ROSSBY, C.-G., 1938: On the mutual adjustment of pressure and velocity distributions in certain simple current systems, II. *Journal of Marine Research,* **1**, 239-263.

ROSSBY, C.-G., 1940: Planetary flow patterns in the atmosphere. *Quarterly Journal of the Royal Meteorological Society,* **66**, 68-87.

SAMELSON, R. M., 1998. Review of: The Earth and the Cosmos (The Legacy of Hans Ertel). *Bulletin of the American Meteorological Society*, **79**, 894-896.

THORPE, A. J., and H. VOLKERT, 1997: Potential vorticity: A short history of its definitions and uses. *Meteorologische Zeitschrift*, N.F. **6**, 275-280.